$c_2\xi \to \infty$ along the real line and the function $J_0(c_2\xi) \to 0$. f_p remains bounded as $P_c \to \infty$ and the eigenfunctions once again reduce to those of the simplified problem, but with the important qualitative difference that the constant a_3 is no longer zero. This difference is crucial, as will be seen in the following argument. As $P_c \to \infty$, one can consider the limit $\xi \to 0$ in such a way that $c_2\xi$ is either zero or has a finite value. In this limit, the term $a_3 J_0(c_2\xi)$ makes a finite contribution to f_p and the eigenfunctions of the exact problem no longer reduce to those of the simplified problem. The implication is that damped disturbances of the simplified eigenvalue problem are not true asymptotic representations of damped disturbances of the exact problem in the limit $P_c \to \infty$. Physically, these arguments imply that, for damped disturbances, there exists a region around $\xi = 0$ of dimension $1/P_c^{1/2}$ where the effects of unsteady mass and energy diffusion and unsteady energy conduction cannot be neglected. For damped disturbances, these effects have a finite contribution even as $P_c \to \infty$ and the "transport region" reduces to a point.

Self-Excited and Damped Disturbances (Large ξ)

In this region it is convenient to write the general solution of Eq. (2) in the form

$$f_p = b_1 H_0^{(1)}(c_1\xi) + b_2 H_0^{(2)}(c_1\xi) + b_3 H_0^{(1)}(c_2\xi) + b_4 H_0^{(2)}(c_2\xi)$$

The requirement that f_p be bounded as $\xi \to \infty$ will imply that any two of the four constants above be zero. The exact choice depends on the arguments of c_1 and c_2. Equation (3) shows that c_1 and c_2 can each be chosen so as to lie above or below the real axis. In particular, c_1 and c_2 can always be chosen so as to lie above the real axis and f_p can be written without loss of generality as

$$f_p = b_1 H_0^{(1)}(c_1\xi) + b_3 H_0^{(1)}(c_2\xi)$$

This form of f_p satisfies the boundary conditions as $\xi \to \infty$. If now the limit $p_c \to \infty$ is considered, $c_2\xi$ will be infinitely large and $H_0^{(1)}(c_2\xi) \to 0$. Thus, for both self-excited and damped oscillations, the eigenfunctions of the exact problem reduce to those of the simplified problem in the limit $P_c \to \infty$ in this region.

Conclusion

All of the plots presented in Ref. 1 can be considered as valid representations in the limit $P_c \to \infty$, provided G_r is interpreted as having only the positive values, i.e., only self-excited disturbances. The simplified formulation can also represent neutral disturbances provided this is obtained as a limit of the self-excited case. No information on damped disturbances can be obtained from the simplified problem.

The full solution of the exact eigenvalue problem for finite values of P_c can be obtained by a straightforward (but very time-consuming) integration of the differential equations using asymptotic solutions developed here for initialization. The eigenvalue and relationships between the arbitrary constants can be obtained by a process of iteration from the matching of the numerical solutions. Such an exercise is not considered worthwhile by the author for two reasons: the needed physical information has already been extracted, and it would be inappropriate to make the stability analysis highly refined when the basic steady-state solution has some mutually contradictory assumptions.[2] A fruitful line of effort in this area would be the inclusion of fluid mechanical nonlinearities and viscosity effects in solutions of the steady-state diffusion flame problem. It is the author's opinion that chemical kinetic nonlinearities are not important in this problem and that the flame surface approximation can be continued to be used in both the steady and unsteady cases.

References

[1]Sheshadri, T. S., "Stability Analysis of a Class of Diffusion Flames," *AIAA Journal*, Vol. 23, Jan. 1985, pp. 88-94.
[2]Williams, F. A., *Combustion Theory*, Addison-Wesley Publishing Co., Reading, MA, 1965, pp. 37-44.
[3]Sheshadri, T. S., "Linearized Unsteady Flame Surface Approximation Result in Complex Notation," *AIAA Journal*, Vol. 21, Dec. 1983, pp. 1770-1772.

The Role of Damping on the Stability of Short Beck's Columns

I. Lottati*
Technion—Israel Institute of Technology
Haifa, Israel

Introduction

THE Bernoulli-Euler theory, which is used extensively in the analysis of dynamic systems that can be approximated by beams, neglects the important effects of deformation due to shear and rotatory inertia. Inclusion of these effects complicated the partial differential equation governing the dynamics of the beam and, thus, a great deal of effort is needed to solve these equations.

The stability of short Beck and Leipholz columns on elastic foundations was studied by Sundararamaiah and Venkateswara Rao[1] using the finite element method.

In the present Note we will confirm the results of Ref. 1, which indicate that for a Timoshenko beam resting on an elastic foundation subjected to a follower force, the elastic foundation has a destabilizing effect on the beam. The effects of viscoelastic and viscous damping on the stability of columns resting on an elastic foundation subjected to a follower force coupled with the effect of shear deformation and rotatory inertia will be examined.

Differential Equations and Boundary Conditions

The coupled equations for the total deflection y and the bending slope ψ for a cantilevered beam subjected to a follower force P are given as (see Ref. 2)

$$I\left(E+E^*\frac{\partial}{\partial t}\right)\frac{\partial^2\psi}{\partial x^2} + sGA\left(\frac{\partial y}{\partial x}-\psi\right) - \rho I\frac{\partial^2\psi}{\partial t^2} = 0$$

$$\rho A\frac{\partial^2 y}{\partial t^2} - sGA\left(\frac{\partial^2 y}{\partial x^2}-\frac{\partial\psi}{\partial x}\right) + P\frac{\partial^2 y}{\partial x^2} + Ky + c\frac{\partial y}{\partial t} = 0 \quad (1)$$

where

E = modulus of elasticity
I = area moment of inertia of cross section
E^* = coefficient of internal dissipation, assumed to be viscoelastic of the Voigt-Kelvin type
G = modulus of rigidity
A = cross-sectional area
s = numerical shape factor for cross section
ρ = density
K = constant elastic foundation modulus
c = viscous damping coefficient

Received Oct. 31, 1984; revision received March 12, 1985.

*Senior Lecturer, Department of Aeronautical Engineering.

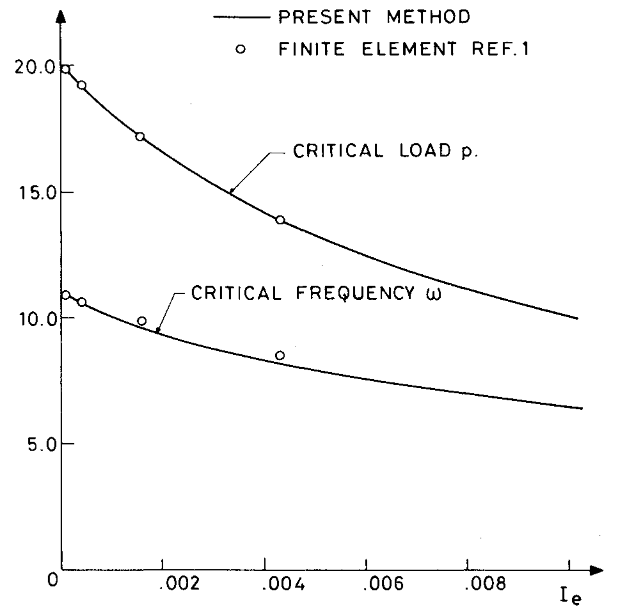

Fig. 1 Variation of the critical load and frequencies due to the change of rotatory inertia and shear deformation.

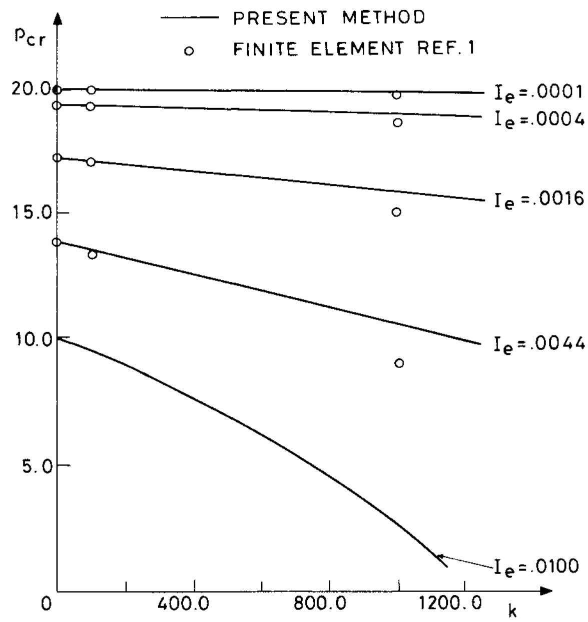

Fig. 2 Variation of the critical load due to the change of the elastic foundation modulus for several values of rotatory inertia.

Introducing nondimensional coordinates ξ,τ results in the following basic partial differential equations:

$$\left(1+\eta\frac{\partial}{\partial\tau}\right)\frac{\partial^2\psi}{\partial\xi^2}+\frac{sGAL^2}{EI}\left(\frac{\partial y}{\partial\xi}-\psi\right)-\frac{I}{AL^2}\frac{\partial^2\psi}{\partial\tau^2}=0$$

$$\frac{\partial^2 y}{\partial\tau^2}-\frac{sGAL^2}{EI}\left(\frac{\partial^2 y}{\partial\xi^2}-\frac{\partial\psi}{\partial\xi}\right)+\frac{PL^2}{EI}\frac{\partial^2 y}{\partial\xi^2}$$

$$+\frac{KL^4}{EI}y+\frac{cL^2}{\sqrt{\rho AEI}}\frac{\partial y}{\partial\tau}=0 \tag{2}$$

where $\tau=\sqrt{EI/\rho AL^4}\,t$, $\xi=x/L$, and $\eta=E^*/E\sqrt{EI/\rho AL^4}$.

Employing standard methods of solution, it is assumed that

$$y=we^{\sigma\tau}e^{r\xi}, \qquad \psi=\varphi e^{\sigma\tau}e^{r\xi} \tag{3}$$

where $\sigma=\alpha+i\omega$; assuming α and ω to be real.

Equation (2) is rewritten as

$$\begin{bmatrix}(1+\eta\sigma)r^2-g_s-\sigma^2 I_e & g_s r\\ g_s r & pr^2+k+\delta\sigma-g_s r^2+\sigma^2\end{bmatrix}\begin{Bmatrix}\varphi\\ w\end{Bmatrix}=0 \tag{4}$$

where

$$g_s=sGAL^2/EI, \quad I_e=I/AL^2, \quad p=PL^2/EI$$

$$k=KL^4/EI, \quad \delta=cL^2/\sqrt{\rho AEI}$$

To get a nontrivial solution to the matrix equation (4), the determinant has to vanish to obtain a fourth-order characteristic equation in r.

Using the standard specification of G, E, and s one can relate g_s and I_e as $g_s=1/\gamma I_e$, where $\gamma=3.2$ was chosen for computing the numerical results. The boundary conditions for this problem are:

$$w=0 \text{ and } \varphi=0 \qquad \text{for } \xi=0$$

$$\frac{\partial\varphi}{\partial\xi}=0 \text{ and } \frac{\partial w}{\partial\xi}-\varphi=0 \qquad \text{for } \xi=1 \tag{5}$$

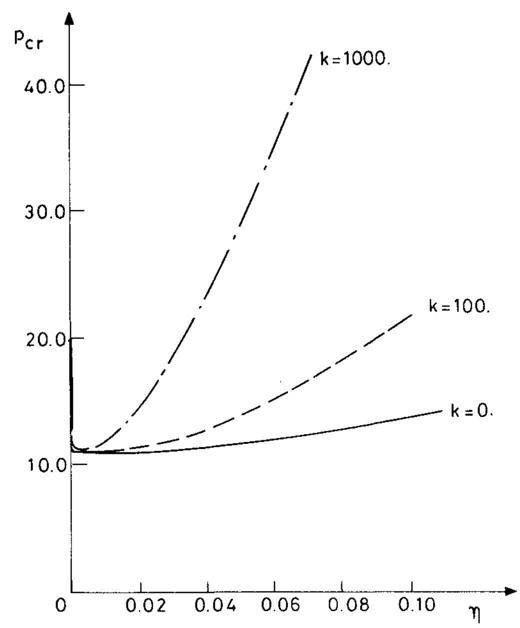

Fig. 3 Influence of the viscoelastic damping coefficient on the critical load for several values of the elastic foundation modulus ($I_e=0.0001$).

The four boundary conditions are reduced to a set of four linear equations. Necessary and sufficient condition for existence of a nontrivial solution is that the determinant of the matrix equation obtained from the formulation of the boundary condition be zero. The combination of ω and p (minimum) that fulfills the characteristic equation and the zero determinant condition rendering the system from neutral stability ($\alpha\leq 0$) to instability ($\alpha>0$) is the condition at which the beam will undergo dynamic instability.

The detailed procedure for solving the problem is stated in Ref. 3.

Applications

Assuming $I_e=0$ and $\eta=\delta=k=0$ the problem is reduced to the usual Bernoulli-Euler (B-E) beam subjected to a follower force. In this case, the critical value of the load is $p=20.05$

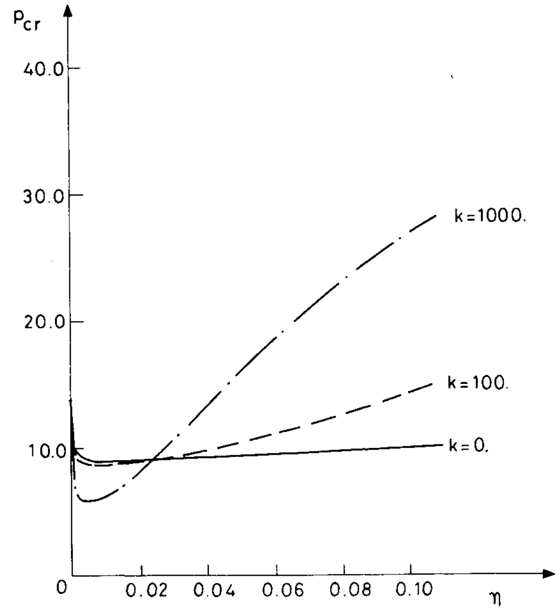

Fig. 4 Influence of the viscoelastic damping coefficient on the critical load for several values of the elastic foundation modulus (I_e = 0.0044).

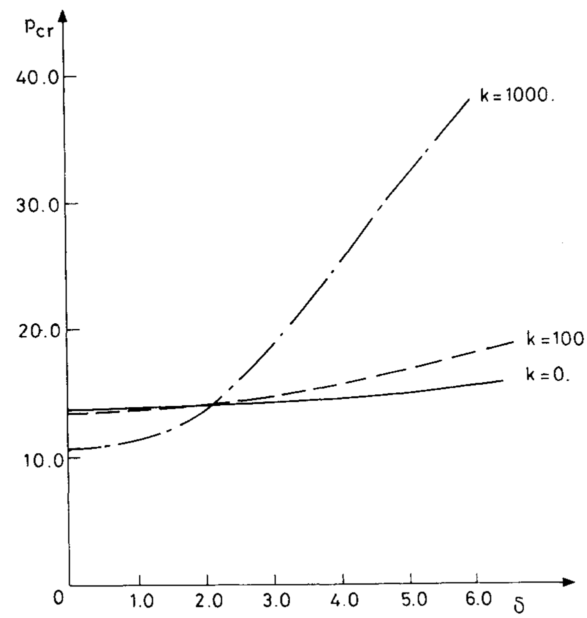

Fig. 5 Influence of the viscous damping coefficient on the critical load for several values of the elastic foundation modulus (I_e = 0.0044).

($\omega = 11.0$), which was first computed by Beck.[4] Starting from the known Beck's solution, one can increase the inertia of the beam (I_e) slowly to get a converged solution for the preceding problems. Figure 1 displays the variation of the critical load and frequency at beam flutter as the inertia of the beam increases. It is seen that the effect of shear deformation and rotatory inertia is to reduce the critical load (destabilizing effect) and frequency at which instability will occur. The critical loads obtained by the present method are in excellent agreement with the results reported in Ref. 1. The frequencies obtained by the finite element method of Ref. 1 are slightly higher than the frequencies computed by the present method (see Fig. 1). Figure 2 shows the variation of the critical load p as a function of increasing the elastic foundation modulus k. The present results confirm the phenomenon reported in Ref. 1 that the elastic foundation reduces the critical load at which the column will undergo dynamic instability.

The effect of viscoelastic damping η coupled with the elastic foundation modulus is plotted in Figs. 3 and 4 for $I_e = 0.0001$ and 0.0044, respectively. The striking phenomenon presented in Figs. 3 and 4 is the sharp reduction in the critical load p for a very lightly damped beam. A similar phenomenon is reported in Ref. 3 for the B-E beam. For shorter beams (higher values of I_e), the reduction in the critical load for very lightly damped beams is more pronounced for higher values of elastic foundation k (see Fig. 4). The results of Fig. 4 show that for lightly damped beams ($\eta < 0.025$) the effect of the elastic foundation is destabilizing, while for higher damping ($\eta > 0.025$) the elastic foundation has a stabilizing effect.

The viscous damping has a stabilizing effect for the B-E beam (small values of I_e), as reported in Ref. 3. Figure 5 shows the influence of the viscous damping coefficient on the critical load for a shorter beam ($I_e = 0.0044$) for several values of the elastic foundation modulus. The results of Figs. 4 and 5 show that for shorter beams and light damping (viscous and viscoelastic), the elastic foundation has a destabilizing effect. Increasing the damping coefficient beyond a certain value causes the elastic foundation to stabilize the system (a higher critical load for a higher elastic foundation coefficient).

References

[1]Sundararamaiah, V. and Venkateswara Rao, G., "Stability of Short Beck and Leipholz Columns on Elastic Foundation," *AIAA Journal*, Vol. 21, Jan. 1983, pp. 1053-1054.

[2]Laithier, B. E. and Paidousis, M. P., "The Equations of Motion of Initially Stressed Timoshenko Tubular Beams Conveying Fluid," *Journal of Sound and Vibration*, Vol. 79, 1981, pp. 175-195.

[3]Lottati, I. and Kornecki, A., "The Effect of an Elastic Foundation and of Dissipative Forces on the Stability of Fluid Conveying Pipes," Technion, Haifa, TAE No. 563, Feb. 1985.

[4]Beck, M., "Die knicklast des einseitig eingespannten, tangential gedruckten stabes," *Zeitschrift fuer Angenwandte Mathematik und Physik*, Vol. 3, 1952, pp. 225-228.

Influence of Mass Representation on the Equations of Motion for Rotating Structures

Robert M. Laurenson*
McDonnell Douglas Astronautics Company
St. Louis, Missouri

Introduction

CONVENTIONAL analysis techniques are not applicable in the case of an elastic structure experiencing significant angular motion. This is of interest because numerous structural configurations such as spinning satellites, rotating shafts, and rotating linkages fall into this category. The analysis of these rotating structures differs from that of stationary structures due to the complexity of the accelerations

Presented as Paper 83-0915 at the AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, Lake Tahoe, NV, May 2-4, 1983; received May 18, 1983; revision received April 26, 1985.

*Section Chief Technology, Structural Dynamics and Loads Department. Senior Member AIAA.